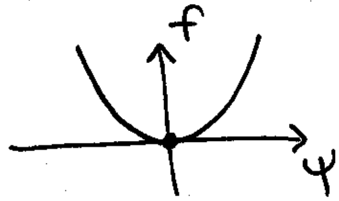
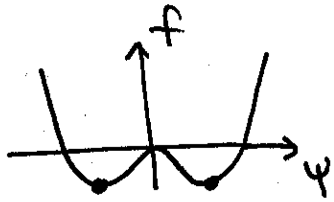
PROBLEM 1

(a) $\boxed{a_4 > 0}$

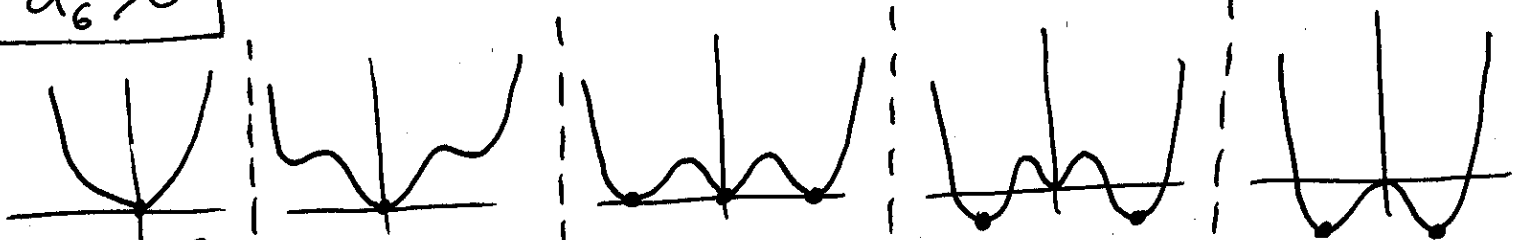




$\boxed{a_2 > 0}$ $\qquad$ $\boxed{a_2 < 0}$

at Equilibrium: $\psi_0 = 0$ $\qquad$ $\psi_0 = \pm\left(-\frac{a_2}{a_4}\right)^{1/2}$

$\boxed{\begin{matrix} a_4 < 0 \\ a_6 > 0 \end{matrix}}$ extrema $\psi_0^2 = 0, \ \frac{-a_4 \pm \sqrt{a_4^2 - 4a_2a_6}}{2a_6}$



$a_2 > \frac{a_4^2}{4a_6}$ | $\frac{3a_4^2}{16a_6} < a_2 < \frac{a_4^2}{4a_6}$ | $a_2 = \frac{3a_4^2}{16a_6}$ | $0 < a_2 < \frac{3a_4^2}{16a_6}$ | $a_2 < 0$

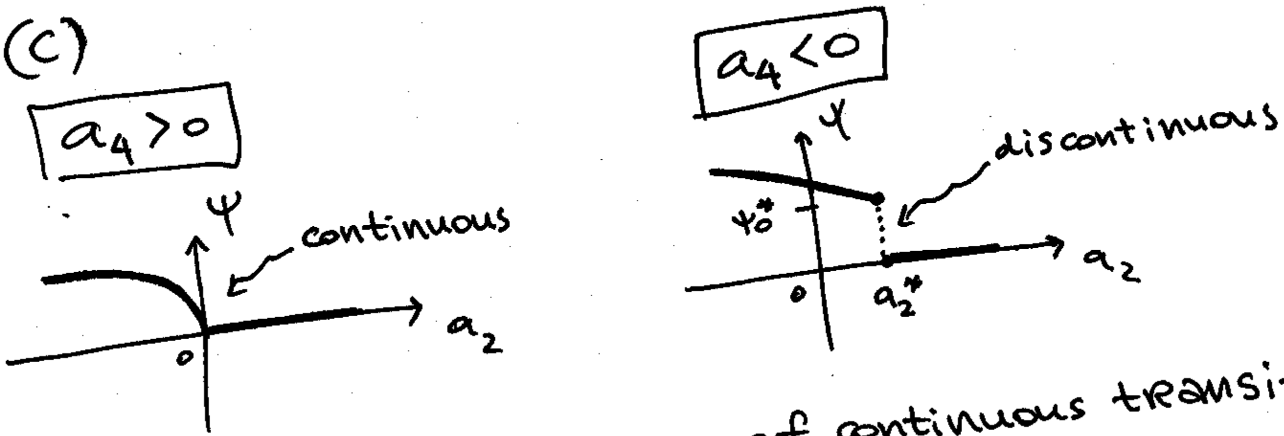
$\underbrace{\hphantom{a_2 > \frac{a_4^2}{4a_6} \quad \frac{3a_4^2}{16a_6} < a_2 < \frac{a_4^2}{4a_6}}}_{\psi_0 = 0}$

$\psi_0 = 0, \ \pm\left(-\frac{3a_4}{4a_6}\right)^{1/2}$ $\qquad$ $\psi_0 = \pm\left(\frac{-a_4 \pm \sqrt{a_4^2 - 4a_2a_6}}{2a_6}\right)^{1/2}$

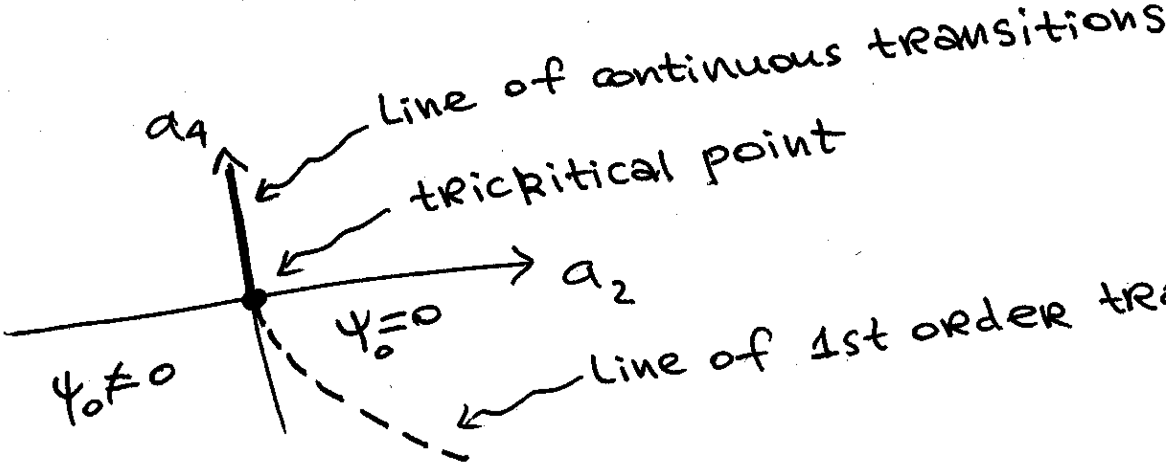
(b) at the transition $f'(\psi_0) = 0$ and $f(\psi_0) = 0$ as can be seen from the middle graph above.

$$\begin{cases} \frac{a_2}{2}\psi_0^2 + \frac{a_4}{4}\psi_0^4 + \frac{a_6}{6}\psi_0^6 = 0 \\ a_2\psi_0 + a_4\psi_0^3 + a_6\psi_0^5 = 0 \end{cases} \Rightarrow \quad \begin{aligned} \psi_0^* &= \pm\left(-\frac{3a_4}{4a_6}\right)^{1/2} \\ a_2^* &= \frac{3}{16}\frac{a_4^2}{a_6} \end{aligned}$$

(c)



(d)



(e) $$f = \frac{a_2}{2}\psi^2 + \frac{a_6}{6}\psi^6 \qquad (a_4 \equiv 0)$$

minimize f: $\quad f' = a_2\psi_0 + a_6\psi_0^5 = 0$

$$\begin{cases} \psi_0^4 = -\dfrac{a_2}{a_6} = \dfrac{c|t|}{a_6} \ , & t < 0 \\ \psi_0 = 0 \ , & t > 0 \end{cases}$$

$$\Rightarrow \psi_0 \sim |t|^{1/4}, \ t < 0, \Rightarrow \boxed{\beta = 1/4}$$

$$f(\psi_0) \sim |t|^{3/2} \quad (\text{use } \psi_0 \sim |t|^{1/4} \text{ in } f)$$

$$C \sim \frac{\partial^2 f}{\partial t^2} \sim |t|^{-1/2} \quad \Rightarrow \quad \boxed{\alpha = 1/2}$$